BIFURCATION ANALYSIS OF DYNAMICAL SYSTEM WITH NONLINEAR DASHPOT AND IMPACTS

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Abstract. A nonlinear viscous damper is a type of damping device used in engineering to dissipate energy and reduce vibrations in structures. Damping is essential in many engineering applications to control the response of structures subjected to dynamic loads, such as earthquakes, wind, or machinery-induced vibrations. In a nonlinear viscous damper, the damping force is not directly proportional to the velocity of the structure, which distinguishes it from linear viscous dampers. The nonlinearity in the damping force-velocity relationship can be designed to provide specific performance characteristics. The main reasons for employing nonlinear viscous dampers include increased energy dissipation – nonlinear viscous dampers can provide higher energy dissipation compared to linear dampers, making them effective in controlling larger vibrations. This work deals with numerical analysis of a single degree of freedom dynamical system representing plate-flow interaction with quadratic drag force subjected to harmonic excitation with and without additional impacts during oscillations. Numerical analysis is based on the bifurcation theory. The theory focuses on understanding the qualitative changes in the behaviour of a system as a parameter is varied. Without additional stoppers the system behaves as a linear system. With "soft" stoppers the system gets limited displacements with the same velocities, multiplicity and more uniform distribution of amplitudes of oscillations. Understanding bifurcations is crucial in predicting and controlling the behaviour of dynamic systems, especially when dealing with nonlinear phenomena.

Keywords: plate-flow interaction, soft impacts, bifurcation analysis.

Introduction

Energy harvesting, the process of capturing and converting ambient energy into usable electrical power, faces several challenges that hinder its widespread adoption and effectiveness. Some of the biggest problems in energy harvesting include: low energy density; variable and unpredictable sources; efficiency and conversion efficiency; miniaturization and integration; environmental constraints; cost and scalability; energy storage and management; standardization and interoperability. Addressing these challenges requires interdisciplinary research efforts, including material science, electrical engineering, mechanical engineering, and computer science. Advances in materials, fabrication techniques, energy conversion technologies, and system design are essential for overcoming these barriers and unlocking the full potential of energy harvesting for sustainable power generation. This paper deals with "plate-flow" system design enhancement.

"Plate-flow nonlinear interaction" typically refers to the interaction between a vibrating plate (such as a structural plate) and a fluid flow (like air or water) surrounding it, where both the plate vibrations and the fluid flow influence each other nonlinearly. This phenomenon is of particular interest in various engineering applications, including aeroelasticity, hydrodynamics, and structural dynamics. Understanding plate-flow nonlinear interaction is crucial for designing structures that can withstand dynamic loading conditions and optimizing control strategies to mitigate adverse effects such as fatigue, vibration-induced noise, or structural failure [1-3].

Analysing plate-flow nonlinear interaction often requires advanced computational tools and experimental techniques, such as computational fluid dynamics (CFD), finite element analysis (FEA), wind tunnel testing, and fluid-structure interaction simulations [4-6]. The method suggested in [7; 8] allows performing tasks of analysis, optimization and synthesis in the interaction of objects with fluids in a simplified way. It is a single-degree-of-freedom system consisting of a plate, spring, energy generator and air flow (1).

$$m\ddot{x} = -cx - b\dot{x} - 1.5BH\rho\left[\left(V\left(0.5 - 0.5\cdot\sin\left(pt\right)\right) + \dot{x}\right)^2\right]\cdot sign\left\{V\left(0.5 - 0.5\cdot\sin\left(pt\right)\right) + \dot{x}\right\}, \tag{1}$$

where m – mass, kg;
x – displacement, m;
$\dot{x}$ – velocity, m·s^{-1};
c – spring stiffness, N·m^{-1};
b – linear generator damping, kg·s^{-1};

V – air flow velocity, m·s^{-1};
p – velocity control actions harmonic angular frequency, s^{-1};
t – time, s;
B – depth of body, m;
H – length of body, m;
ρ – air density, kg·m^{-3}.

Drag force is nonlinear as depends on relative plate-flow velocity square. Bifurcation analysis with p as the bifurcation parameter is done. Then, soft impacts were added to the system in form of two stoppers for additional mitigation of vibrations. So, the restoring force became piecewise linear (2)

$$f(x) = \begin{cases} c_1 x - (c_1 - c) \cdot d_1 \ if \ x \le d_1 \\ cx \ if \ d_1 < x \le d_2 \\ c_1 x - (c_1 - c) \cdot d_2 \ if \ x > d_2 \end{cases}, \quad (2)$$

where c_1 – left/right stopper stiffness, N·m^{-1};
d_1 – left stopper position, m;
d_2 – right stopper position, m.

Graphical representation of restoring forces considered in this paper is given in Fig. 1.

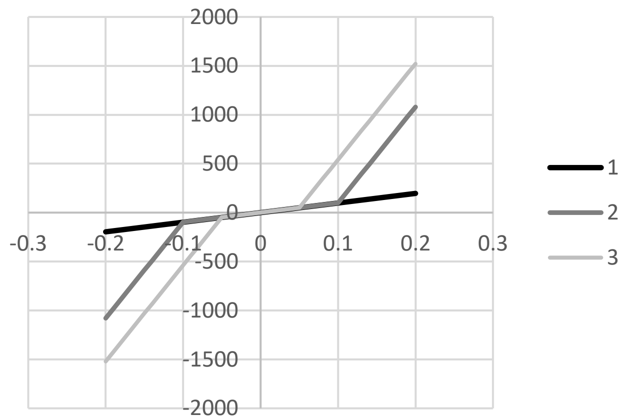

Fig. 1. **Restoring force graphs:** 1 – linear, no stoppers; 2 – piecewise linear, stopper position ± 0.1 m; 3 – piecewise linear, stopper position ± 0.05 m

Then bifurcation analysis was repeated for these two piecewise linear systems. Theory and the results are presented in the following sections.

Theory and methodology

Bifurcation analysis was done in Spring software [9] which is based on the theory of complete bifurcation groups [10]. The theory of complete bifurcation groups is a mathematical framework used to study bifurcations in dynamical systems, particularly in the context of differential equations. The bifurcation theory deals with the qualitative changes in the behaviour of a system as parameters are varied, and complete bifurcation groups provide a systematic way to understand and classify these changes comprehensively. In many dynamical systems, bifurcations occur in families, where different types of bifurcations can coexist and interact with each other. A complete bifurcation group is a set of all possible bifurcations that can occur for a given family of dynamical systems as parameters are varied. A feature of the approach used by the authors is the study, along with stable solutions, of unstable ones, which makes it possible to find the so-called rare attractors. The theory of complete bifurcation groups provides a powerful tool for understanding the rich variety of behaviours exhibited by dynamical systems as parameters are varied.

Results and discussion

Bifurcation analysis was done for the following system parameter set: m = 10 kg, c = 981 N·m^{-1}, c_1 = 9810 N·m^{-1}, b = 10 kg·s^{-1}, V = 10 m·s^{-1}, B = H = 1 m, $d1$ = -0.05 m and -0.1 m, $d2$ = 0.05 m and

0.1 m, ρ = 1.25 kg·m^{-3}, p = var. So, p was taken as a parameter to be varied. Results of bifurcation analysis are presented in Fig. 2 in form of amplitude response of the system vs velocity control actions harmonic angular frequency. Curve 1 corresponds to the nonlinear drag force and linear restoring force. In this case the bifurcation diagram is similar to a linear system – each parameter p value has only one possible periodic solution P1 with resonance at $p \approx 9.7$ s^{-1}.

Curve 2 and Curve 3 represent bifurcation diagrams for the nonlinear drag force and piecewise linear restoring forces with stoppers at ± 0.1 m and ± 0.05 m correspondingly. In both cases amplitudes of oscillations in the resonant zone are lower with respect to ones corresponding to the system with the linear restoring force. The frequency response curves of piecewise linear systems slope downward what is common for nonlinear systems. As a result, at the p parameter range from 16 to 19.5 s^{-1} for the Curve 2 and from 20 to 24 s^{-1} for the Curve 3 one can observe phenomena of multiplicity – there are two possible stable periodic solutions P1 with higher and lower amplitudes. For these ranges additional study of basins of attraction are needed.

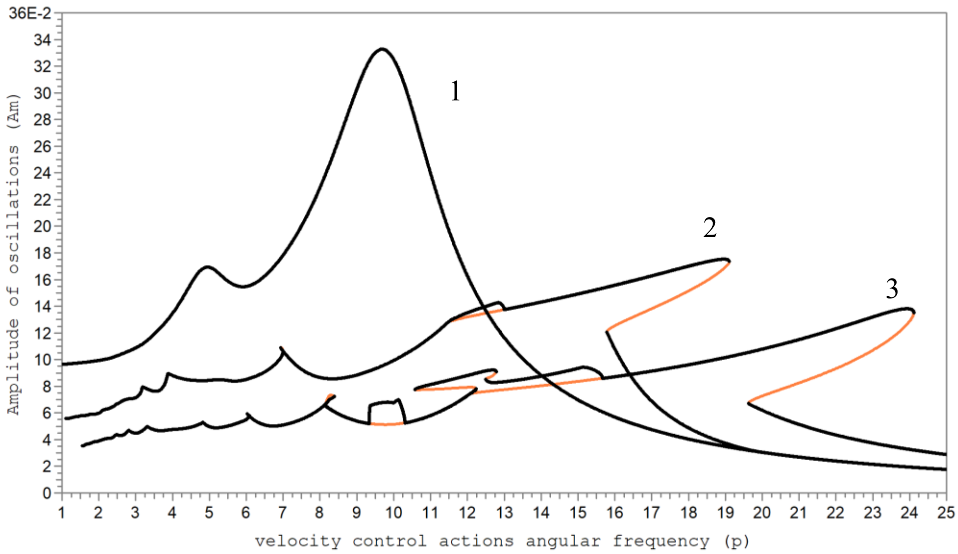

Fig. 2. **Bifurcation diagrams:** 1 – system with linear restoring force, no stoppers; 2 – system with piecewise linear restoring force, stoppers at ± 0.1 m; 3 – system with piecewise linear restoring force, stoppers at ± 0.05 m (black colour – stable solutions; red colour – unstable solutions)

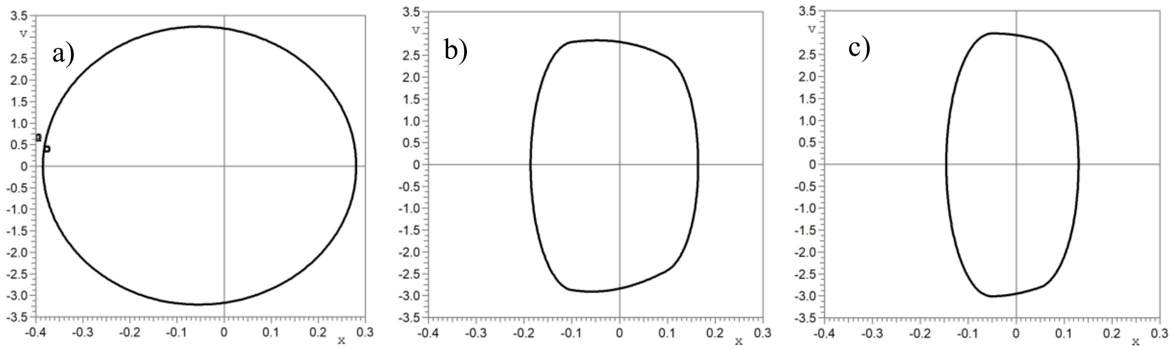

Fig. 3. **Phase trajectories of resonant solutions:** a – system with linear restoring force, p = 9.7 s^{-1}; b – system with piecewise linear restoring force and stoppers at ± 0.1 m, p = 19 s^{-1}; c – system with piecewise linear restoring force and stoppers at ± 0.05 m, p = 24 s^{-1}

Additionally, for Curve 3, periodic solution P1 is unstable in the range of p from 12 to 16 s^{-1}. Here the stable periodic solution P2 is obtained. So, the presence of unstable solutions does not lead to the appearance of chaotic behavior of the system – in the studied range of the parameter p, one or two stable solutions correspond to each frequency. Phase trajectories of resonant periodic solutions for three cases of the restoring force are presented in Fig.3. So, displacements are limited by stoppers but velocities are almost the same.

Another interesting note can be made from the results of the study. As it can be observed in Fig. 2, the amplitudes of oscillations “distributed” more uniformly in cases of systems with piecewise linear restoring forces along all presented parameter *p* ranges.

Conclusions

1. The plate-flow interaction system is studied, which allows performing tasks of analysis, optimization and synthesis in the interaction of objects with fluids in a simplified way.
2. If only the drag force is nonlinear dependent on the relative plate-flow velocity square, dynamical behaviour of the system is similar to one with nonlinearities.
3. Adding the “soft” stoppers makes the system nonlinear. This limits the amplitude of oscillations, but velocities are almost the same. And such typical for nonlinear systems phenomenon as multiplicity is obtained.
4. The amplitudes of oscillations “distributed” more uniformly in cases of systems with piecewise linear restoring forces along all studied range of the parameter *p* – velocity control actions angular frequency.

Author contributions

Conceptualization, J.V.; methodology, S.S. and V.J.; software, S.S. and V.J.; formal analysis, S.S; investigation, S.S.; writing – original draft preparation, S.S.; writing – review and editing, S.S.; funding acquisition, J.V. All authors have read and agreed to the published version of the manuscript.

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